
Computer Graphics III – Radiometry

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Direction, solid angle, spherical integrals

Direction in 3D

- **Direction** = unit vector in 3D

- Cartesian coordinates

$$\omega = [x, y, z], \quad x^2 + y^2 + z^2 = 1$$

- Spherical coordinates

$$\omega = [\theta, \varphi]$$

$$\theta \in [0, \pi]$$

$$\varphi \in [0, 2\pi]$$

$$\theta = \arccos z$$

$$\varphi = \arctan \frac{y}{x}$$

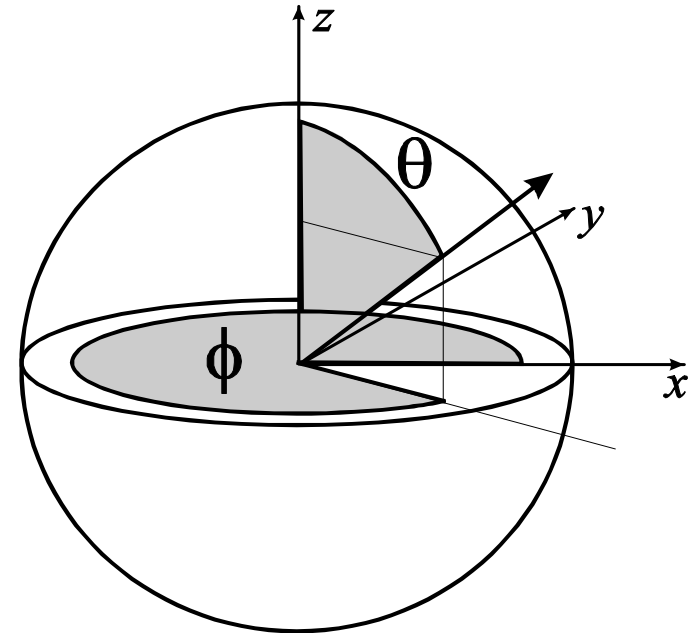
$$x = \sin \theta \cos \varphi$$

$$y = \sin \theta \sin \varphi$$

$$z = \cos \theta$$

- θ ... *polar angle* – angle from the Z axis

- ϕ ... *azimuth* – angle measured counter-clockwise from the X axis



Function on a unit sphere

- Function as any other, except that its argument is a direction in 3D
- Notation
 - $F(\omega)$
 - $F(x,y,z)$
 - $F(\theta,\phi)$
 - ...
 - Depends in the chosen representation of directions in 3D

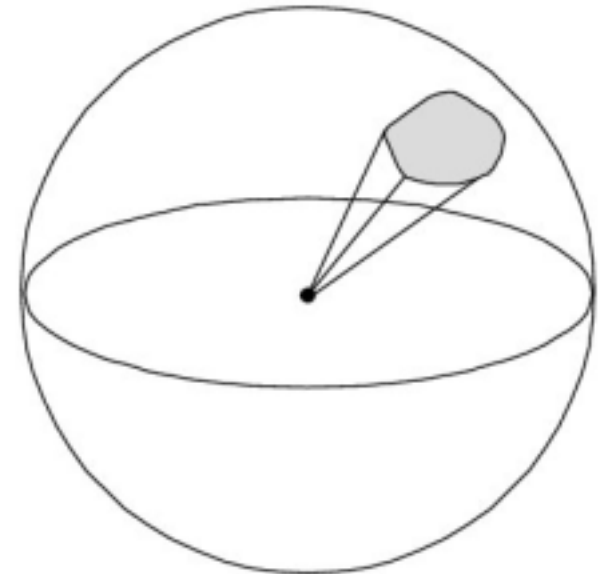
Solid angle

■ Planar angle

- Arc length on a unit circle
- A full circle has 2π radians (unit circle has the length of 2π)

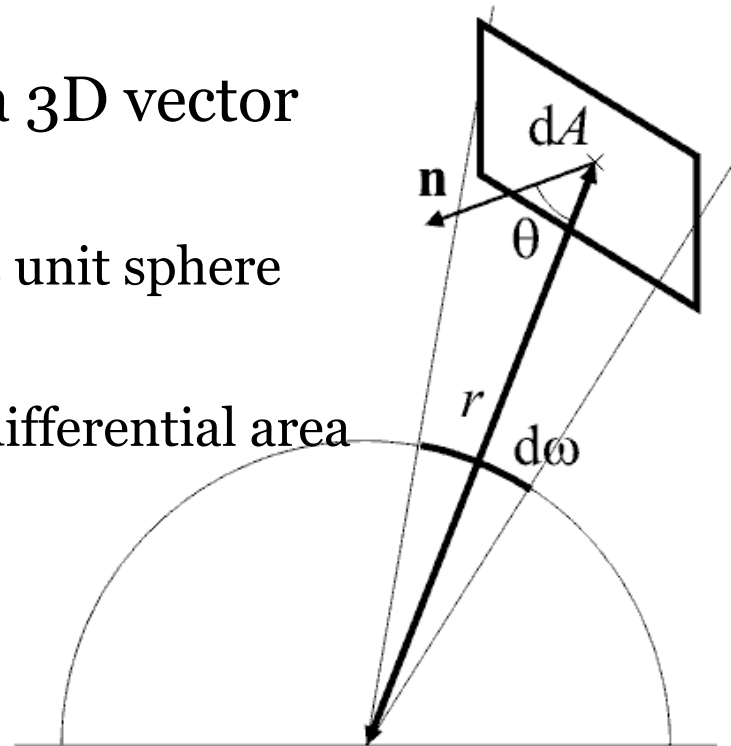
■ Solid angle (steradian, sr)

- Surface area on an unit sphere
- Full sphere has 4π steradians



Differential solid angle

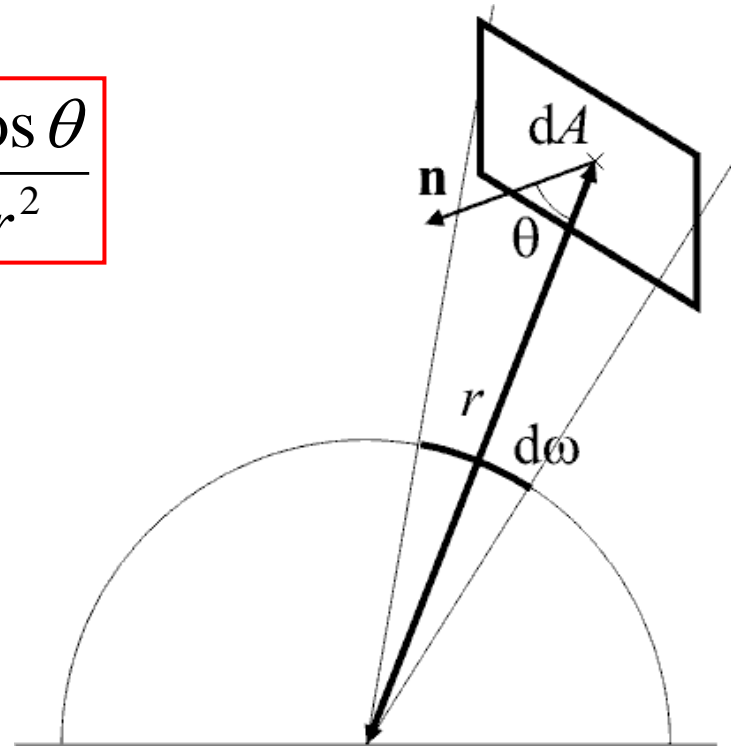
- “Infinitesimally small” solid angle around a given direction
- By convention, represented as a 3D vector
 - Magnitude ... $d\omega$
 - Size of a differential area on the unit sphere
 - Direction ... ω
 - Center of the projection of the differential area on the unit sphere



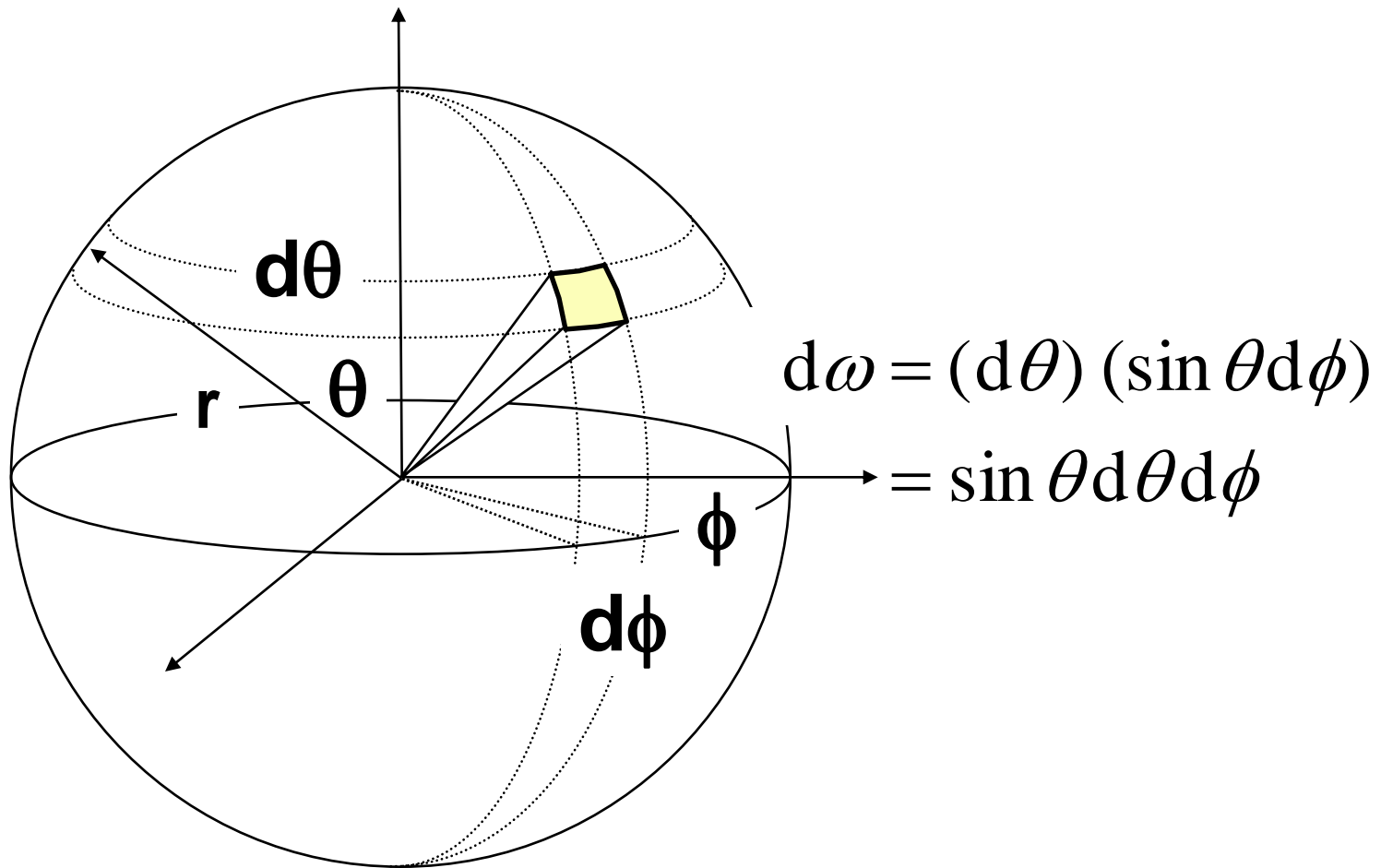
Differential solid angle

- (Differential) solid angle subtended by a differential area

$$d\omega = dA \frac{\cos \theta}{r^2}$$



Differential solid angle



Radiometry and photometry

Radiometry and photometry

- “**Radiometry** is a set of techniques for measuring electromagnetic radiation, including visible light.
- Radiometric techniques in optics characterize the distribution of the radiation's power in space, as opposed to **photometric** techniques, which characterize the light's interaction with the human eye.”

(Wikipedia)

Radiometry and photometry

■ Radiometric quantities

- Radiant energy
(zářivá energie) – Joule
- Radiant flux
(zářivý tok) – Watt
- Radiant intensity
(zářivost) – Watt/sr
- *Denoted by subscript e*

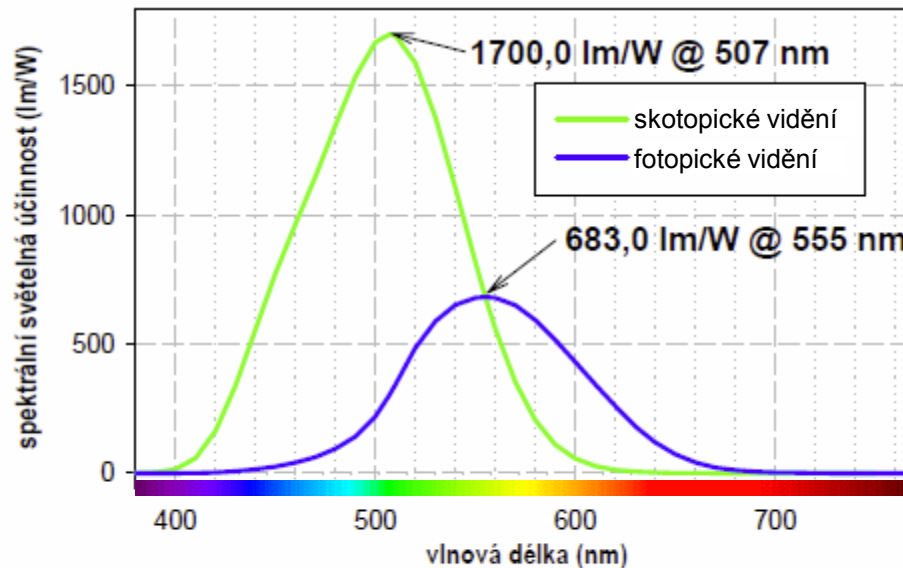
■ Photometric quantities

- Luminous energy
(světelná energie) –
Lumen-second, a.k.a.
Talbot
- Luminous flux
(světelný tok) – Lumen
- Luminous intensity
(svítivost) – candela
- *Denoted by subscript v*

Relation between photo- and radiometric quantities

■ Spectral luminous efficiency $K(\lambda)$

$$K(\lambda) = \frac{d\Phi_{\lambda}}{d\Phi_{e\lambda}}$$



Obr. 7. Spektrální světelná účinnost při fotopickém (denním) vidění a skotopickém (soumrakovém) vidění.

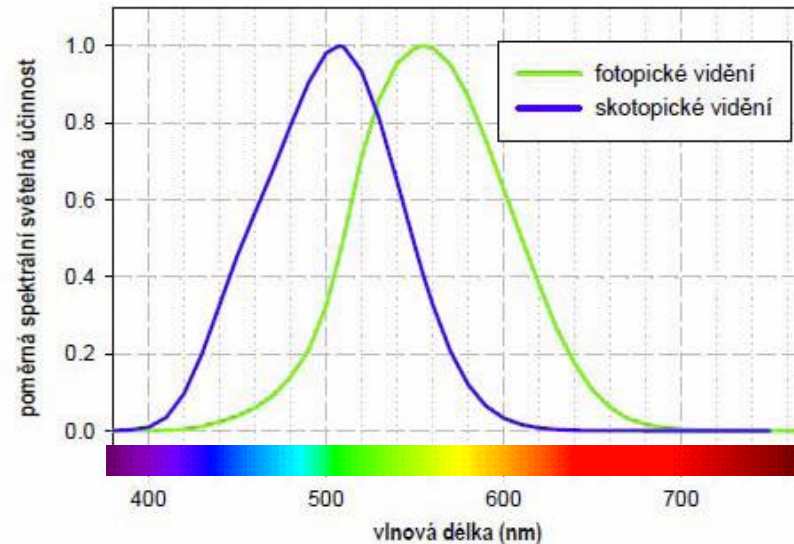
Relation between photo- and radiometric quantities

- Visual response to a spectrum:

$$\Phi = \int_{380\text{nm}}^{770\text{nm}} K(\lambda) \Phi_e(\lambda) d\lambda$$

Relation between photo- and radiometric quantities

- **Relative spectral luminous efficiency $V(\lambda)$**
 - Sensitivity of the eye to light of wavelength λ relative to the peak sensitivity at $\lambda_{\max} = 555 \text{ nm}$ (for photopic vision).
 - CIE standard 1924



Obr. 6. Poměrná spektrální světelná účinnost při fotopickém (denním) a skotopickém (soumrakovém) vidění.

Relation between photo- and radiometric quantities

■ Radiometry

- More fundamental – photometric quantities can all be derived from the radiometric ones

■ Photometry

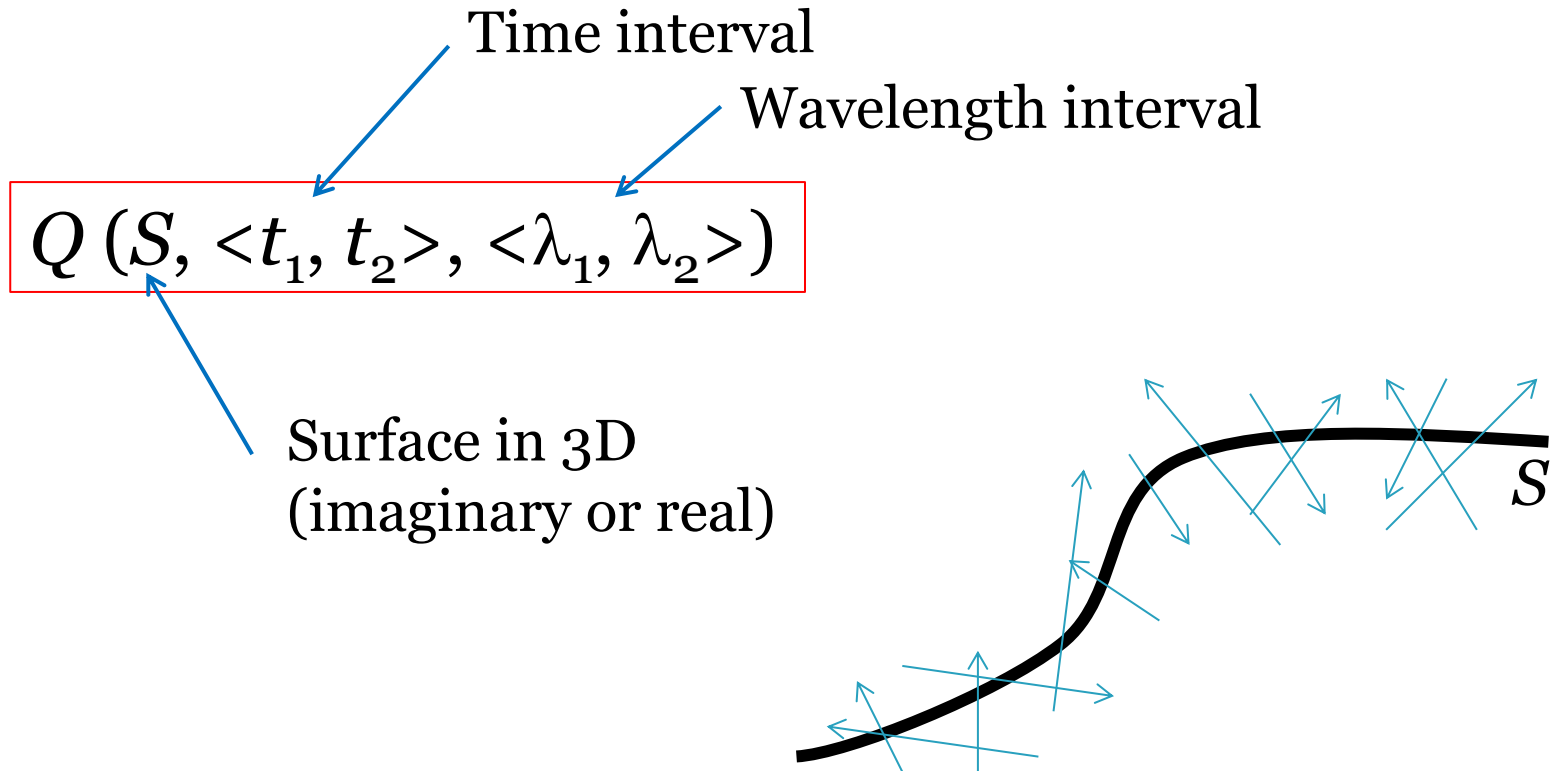
- Longer history – studied through psychophysical (empirical) studies long before Maxwell equations came into being.

Radiometric quantities

Transport theory

- Empirical theory describing flow of “energy” in space
- **Assumption:**
 - Energy is continuous, infinitesimally divisible
 - Needs to be taken so we can use derivatives to define quantities
- Intuition of the “energy flow”
 - Particles flying through space
 - No mutual interactions (implies linear superposition)
 - Energy density proportional to the density of particles
 - This intuition is abstract, empirical, and has nothing to do with photons and quantum theory

Radiant energy – Q [J]



- **Unit:** Joule, J

Spectral radiant energy – Q [J]

- Energy of light at a specific wavelength
 - „Density of energy w.r.t wavelength“

$$Q_\lambda(S, \langle t_1, t_2 \rangle, \lambda) = \lim_{\substack{d(\lambda_1, \lambda_2) \rightarrow 0 \\ \lambda \in \langle \lambda_1, \lambda_2 \rangle}} \frac{Q(S, \langle t_1, t_2 \rangle, \langle \lambda_1, \lambda_2 \rangle)}{\mu \langle \lambda_1, \lambda_2 \rangle} = \text{formally} = \frac{dQ}{d\lambda}$$

- We will leave out the subscript and argument λ for brevity
 - We always consider spectral quantities in image synthesis
- **Photometric quantity:**
 - Luminous energy, unit Lumen-second aka Talbot

Radiant flux (power) – Φ [W]

- How quickly does energy „flow“ from/to surface S ?
 - „Energy density w.r.t. time“

$$\Phi(S, t) = \lim_{\substack{d\langle t_1, t_2 \rangle \rightarrow 0 \\ t \in \langle t_1, t_2 \rangle}} \frac{Q(S, \langle t_1, t_2 \rangle)}{\mu\langle t_1, t_2 \rangle} = (\text{formálně}) = \frac{dQ}{dt}$$

- **Unit:** Watt – W
- **Photometric quantity:**
 - Luminous flux, unit Lumen

Irradiance— E [$\text{W}\cdot\text{m}^{-2}$]

- What is the spatial flux density at a given point \mathbf{x} on a surface S ?

$$E(\vec{x}) = \lim_{\substack{d(S) \rightarrow 0 \\ \vec{x} \in S, S \subseteq P}} \frac{\Phi_i(S)}{\mu(S)} = (\text{formálně}) = \frac{d\Phi_i}{dS}$$

- Always defined w.r.t some point \mathbf{x} on S with a specified surface normal $N(\mathbf{x})$.
 - **Irradiance DOES depend on $N(\mathbf{x})$** (Lambert law)
- We're only interested in light arriving from the “outside” of the surface (given by the orientation of the normal).

Irradiance – E [$W.m^{-2}$]

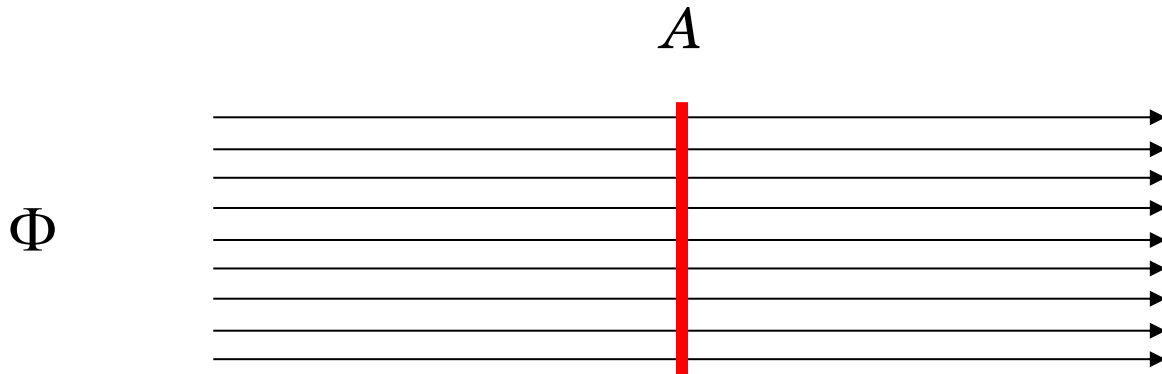
- **Unit:** Watt per meter squared – $W.m^{-2}$
- **Photometric quantity:**
 - Illuminance, unit Lux = $lumen.m^{-2}$

light meter
(cz: expozimetr)



Lambert cosine law

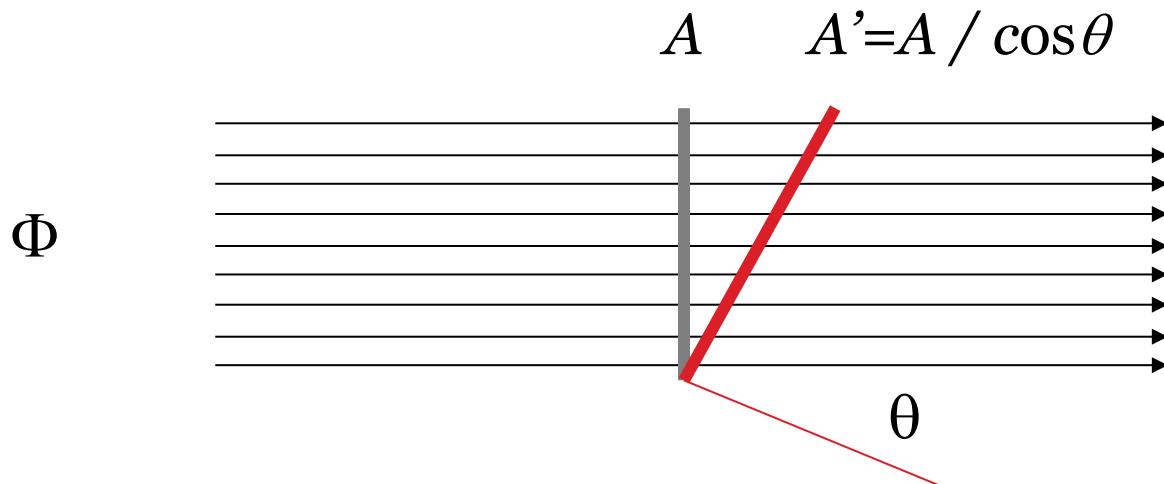
- Johan Heinrich Lambert, *Photometria*, 1760



$$E = \frac{\Phi}{A}$$

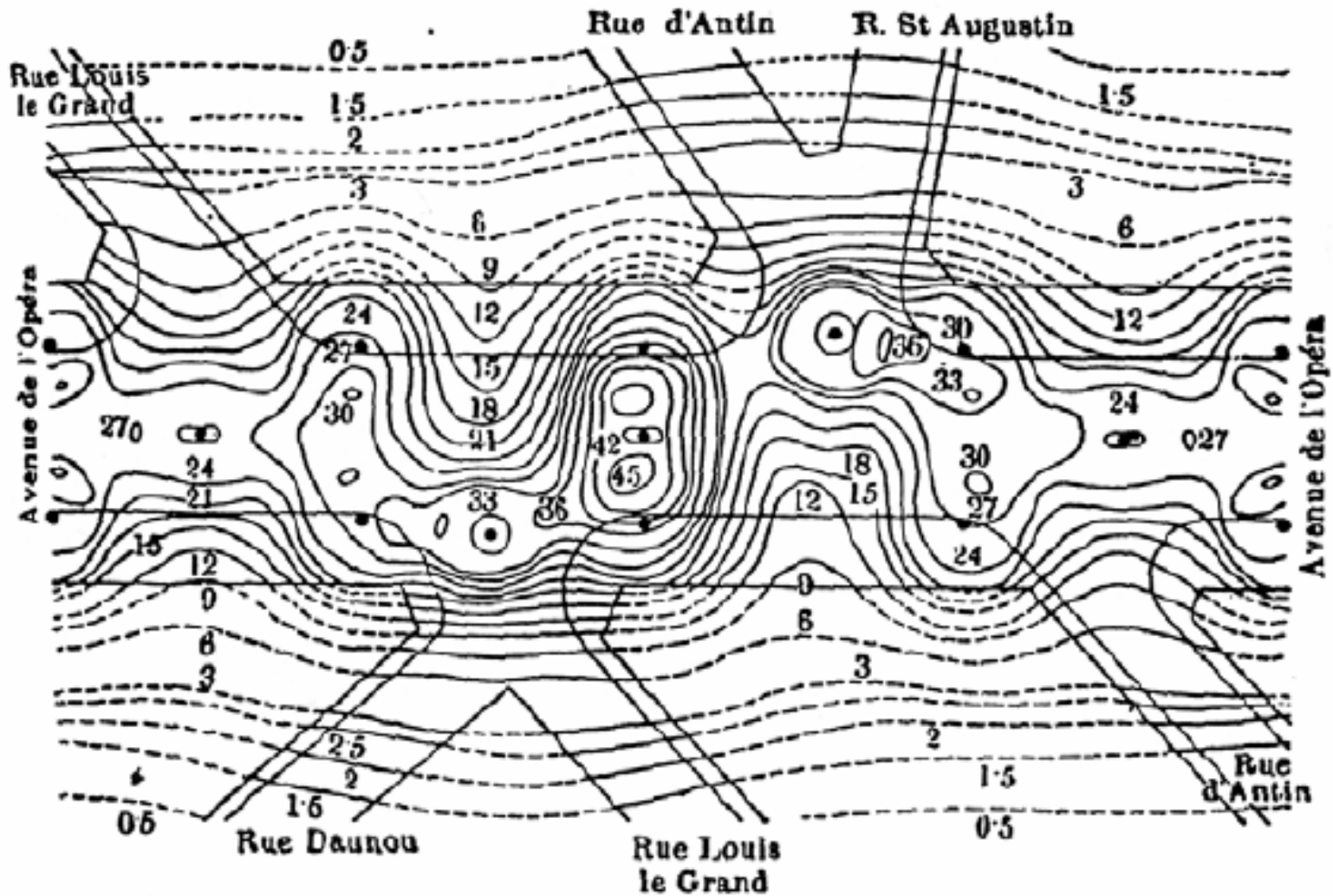
Lambert cosine law

- Johan Heinrich Lambert, *Photometria*, 1760



$$E' = \frac{\Phi}{A'} = \frac{\Phi}{A} \cos \theta$$

Irradiance Map or Light Map



Isolux contours

Typical Values of Illuminance [lm/m^2]

Sunlight plus skylight	100,000 lux
Sunlight plus skylight (overcast)	10,000
Interior near window (daylight)	1,000
Artificial light (minimum)	100
Moonlight (full)	0.02
Starlight	0.0003

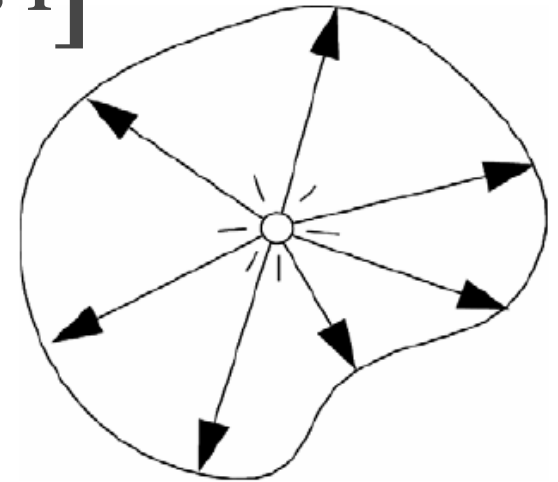
Radiant exitance – B [$\text{W}\cdot\text{m}^{-2}$]

- Same as irradiance, except that it describes *exitant* radiation.
 - The exitant radiation can either be directly emitted (if the surface is a light source) or reflected.
- **Common name: radiosity**
- **Denoted: B , M**
- **Unit: Watt per meter squared – $\text{W}\cdot\text{m}^{-2}$**
- **Photometric quantity:**
 - Luminosity, unit Lux = $\text{lumen}\cdot\text{m}^{-2}$

Radiant intensity – I [$\text{W}\cdot\text{sr}^{-1}$]

- Angular flux density in direction ω

$$I(\omega) = \frac{d\Phi(\omega)}{d\omega}$$



- **Definition:** Radiant intensity is the power per unit solid angle emitted by a point source.
- **Unit:** Watt per steradian – $\text{W}\cdot\text{sr}^{-1}$
- **Photometric quantity**
 - Luminous intensity,
unit Candela ($\text{cd} = \text{lumen}\cdot\text{sr}^{-1}$), **SI base unit**

Point light sources

- Light emitted from a single point
 - Mathematical idealization, does not exist in nature
- Emission completely described by the radiant intensity as a function of the direction of emission: $I(\omega)$
 - **Isotropic point source**
 - Radiant intensity independent of direction
 - **Spot light**
 - Constant radiant intensity inside a cone, zero elsewhere
 - **General point source**
 - Can be described by a **goniometric diagram**
 - Tabulated expression for $I(\omega)$ as a function of the direction ω
 - Extensively used in illumination engineering

Spot Light

- Point source with a directionally-dependent radiant intensity
- Intensity is a function of the deviation from a reference direction \mathbf{d} :

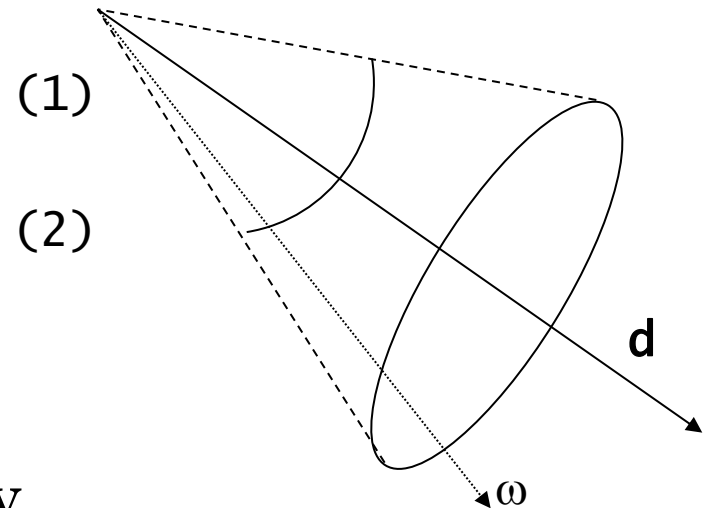
$$I(\omega) = f(\angle\omega, \mathbf{d})$$

- E.g.

$$I(\omega) = I_o \cos \angle(\omega, \mathbf{d}) = I_o (\omega \cdot \mathbf{d})$$

$$I(\omega) = \begin{cases} I_o & \angle(\omega, \mathbf{d}) < \tau \\ 0 & \text{otherwise} \end{cases}$$

- What is the total flux emitted by the source in the cases (1) a (2)?
(See exercises.)



Light Source Goniometric Diagrams

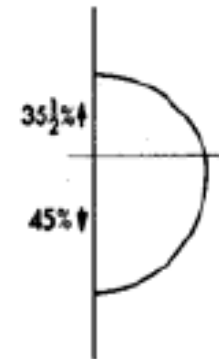
3



Porcelain-enameled ventilated standard dome with incandescent lamp



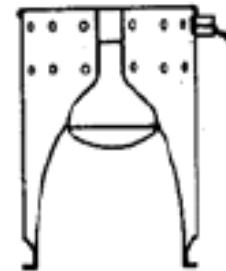
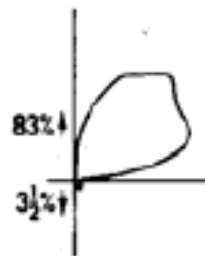
Pendant diffusing sphere with incandescent lamp



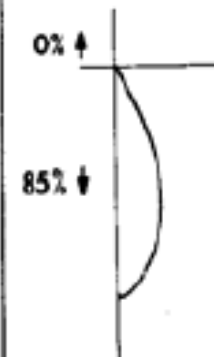
2



Concentric ring unit with incandescent silvered-bowl lamp



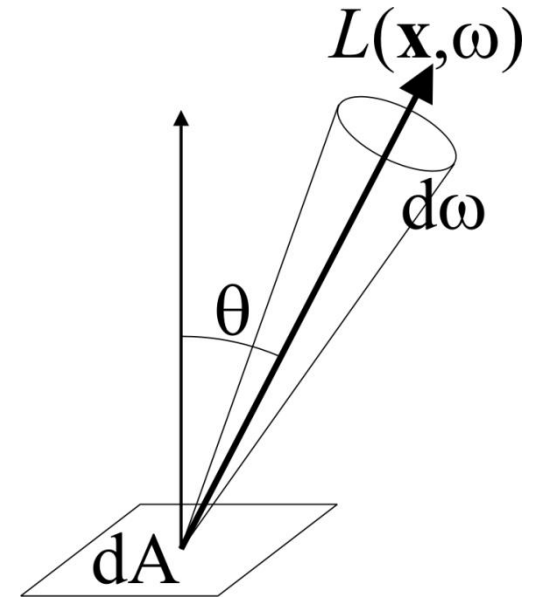
R-40 flood with specular anodized reflector skirt; 45° cutoff



Radiance – L [$\text{W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}$]

- Spatial and directional flux density at a given location \mathbf{x} and direction ω .

$$L(\mathbf{x}, \omega) = \frac{d^2\Phi}{\cos\theta dA d\omega}$$

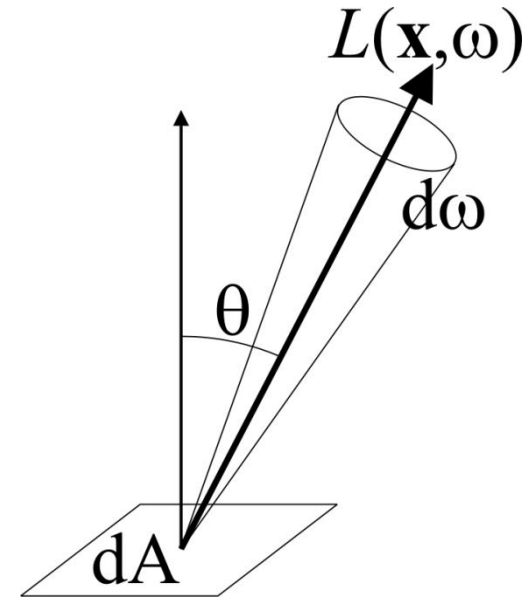


- **Definition:** *Radiance* is the power per unit area **perpendicular to the ray** and per unit solid angle in the direction of the ray.

Radiance – L [$\text{W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}$]

- Spatial and directional flux density at a given location \mathbf{x} and direction ω .

$$L(\mathbf{x}, \omega) = \frac{d^2\Phi}{\cos\theta dA d\omega}$$



- **Unit:** $\text{W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}$
- **Photometric quantity**
 - Luminance, unit **candela.m⁻²** (a.k.a. Nit – used only in English)

The cosine factor $\cos \theta$ in the definition of radiance

- $\cos \theta$ compensates for the decrease of irradiance with increasing θ
 - The idea is that **we do not want** radiance to depend on the mutual orientation of the ray and the reference surface
- If you illuminate some surface while rotating it, then:
 - **Irradiance does change with the rotation** (because the actual spatial flux density changes).
 - **Radiance does not change** (because the flux density change is exactly compensated by the $\cos \theta$ factor in the definition of radiance). And that's what we want.

Typical Values of Luminance [cd/m²]

Surface of the sun	2,000,000,000 nit
Sunlight clouds	30,000
Clear day	3,000
Overcast day	300
Moon	0.03

The Sky Radiance Distribution



Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

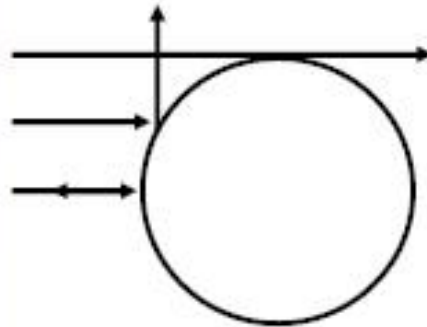


Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

From Greenler, Rainbows, halos and glories

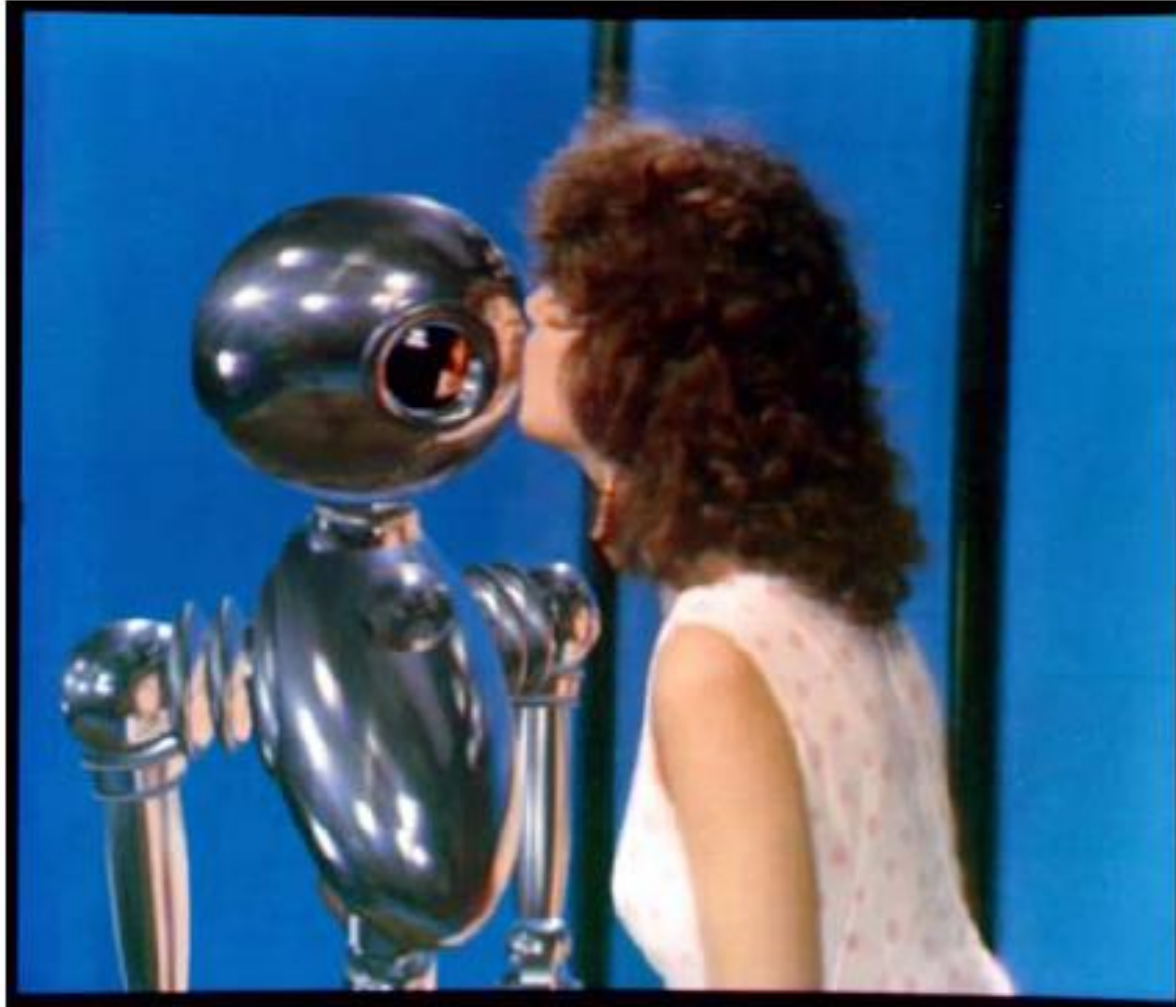
Gazing Ball \Rightarrow Environment Maps

Miller and Hoffman, 1984



- Photograph of mirror ball
- Maps all spherical directions to a to circle
- Reflection direction indexed by normal
- Resolution function of orientation

Environment Maps



Interface, Chou and Williams (ca. 1985)

Calculation of the remaining quantities from radiance

$$E(\mathbf{x}) = \int_{H(\mathbf{x})} L(\mathbf{x}, \omega) \cos \theta \, d\omega$$

$$\begin{aligned} \Phi &= \int_A E(\mathbf{x}) \, dA_{\mathbf{x}} \\ &= \int_A \int_{H(\mathbf{x})} L(\mathbf{x}, \omega) \cos \theta \, d\omega \, dA_{\mathbf{x}} \end{aligned}$$

$\cos \theta \, d\omega =$ projected solid angle

$H(\mathbf{x}) =$ hemisphere above the point \mathbf{x}

Area light sources

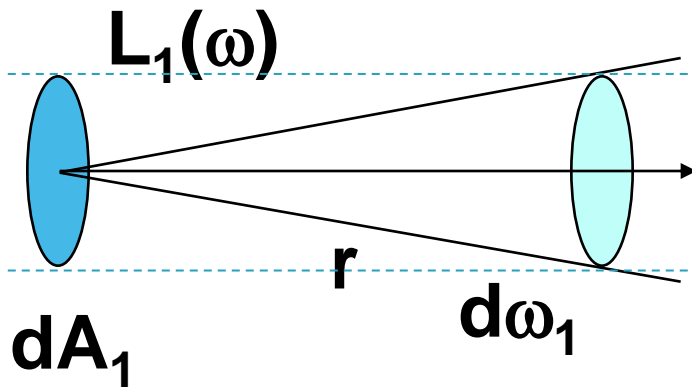
- Emission of an area light source is fully described by the emitted radiance $L_e(\mathbf{x}, \omega)$ for all positions on the source \mathbf{x} and all directions ω .
- The total emitted power (flux) is given by an integral of $L_e(\mathbf{x}, \omega)$ over the surface of the light source and all directions.

$$\Phi = \int_A \int_{H(\mathbf{x})} L_e(\mathbf{x}, \omega) \cos \theta \, d\omega \, dA$$

Properties of radiance (1)

- **Radiance is constant along a ray in vacuum**
 - ❑ Fundamental property for light transport simulation
 - ❑ This is why radiance is the quantity associated with rays in a ray tracer
 - ❑ Derived from energy conservation (next two slides)

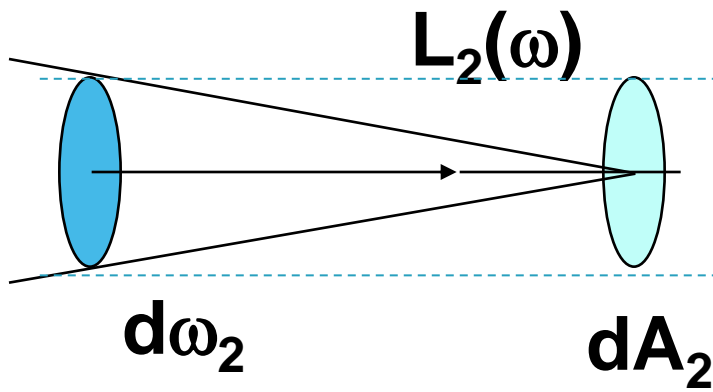
Energy conservation along a ray



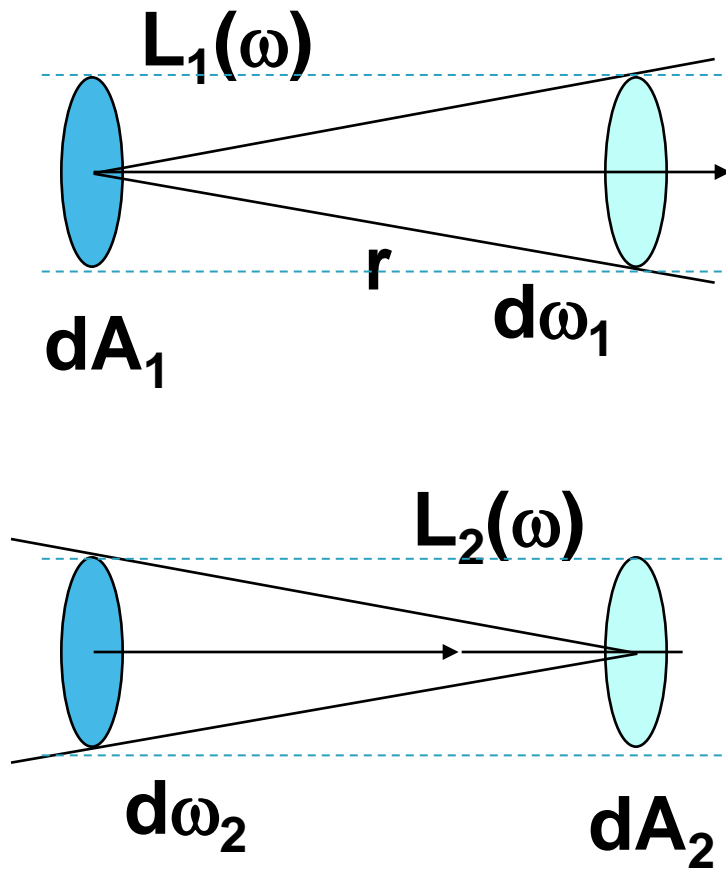
$$L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2$$

emitted
flux

received
flux



Energy conservation along a ray



$$L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2$$

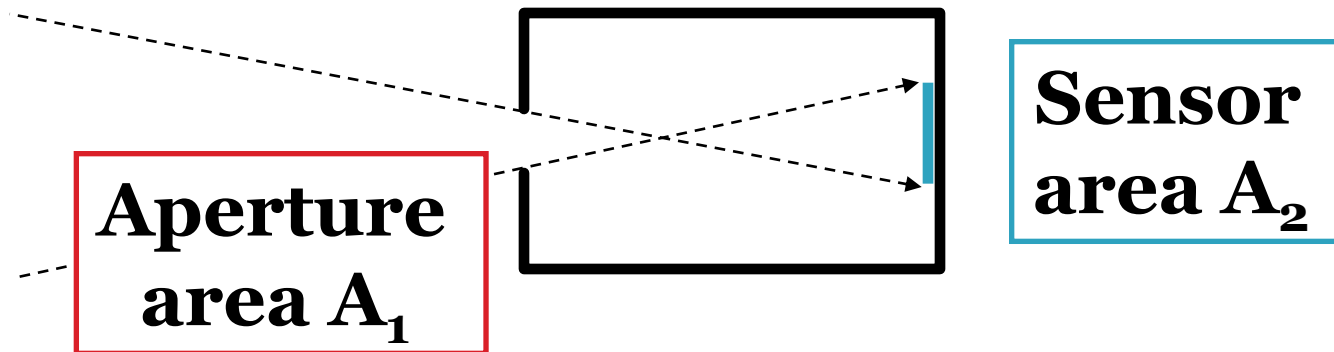
$$\begin{aligned} \underline{T} &= d\omega_1 dA_1 = d\omega_2 dA_2 = \\ &= \frac{dA_1 dA_2}{r^2} \end{aligned}$$

ray throughput

$$L_1 = L_2$$

Properties of radiance (2)

- **Sensor response** (i.e. camera or human eye) is directly proportional to the value of **radiance** reflected by the surface visible to the sensor.



$$\underline{R} = \int_{A_2} \int_{\Omega} L_{in}(\mathbf{A}, \omega) \cdot \cos \theta \, d\omega \, dA = \underline{L_{in} \cdot T}$$

Incoming / outgoing radiance

- Radiance is **discontinuous** at an interface between materials
 - Incoming radiance – $L^i(\mathbf{x}, \omega)$
 - radiance just before the interaction (reflection/transmission)
 - Outgoing radiance – $L^o(\mathbf{x}, \omega)$
 - radiance just after the interaction

Radiometric and photometric terminology

Fyzika <i>Physics</i>	Radiometrie <i>Radiometry</i>	Fotometrie <i>Photometry</i>
Energie <i>Energy</i>	Zářivá energie <i>Radiant energy</i>	Světelná energie <i>Luminous energy</i>
Výkon (tok) <i>Power (flux)</i>	Zářivý tok <i>Radiant flux (power)</i>	Světelný tok (výkon) <i>Luminous power</i>
Hustota toku <i>Flux density</i>	Ozáření <i>Irradiance</i>	Osvětlení <i>Illuminance</i>
dtto	Intenzita vyzařování <i>Radiosity</i>	??? <i>Luminosity</i>
Úhlová hustota toku <i>Angular flux density</i>	Zář <i>Radiance</i>	Jas <i>Luminance</i>
??? Intensity	Zářivost <i>Radiant Intensity</i>	Svítivost <i>Luminous intensity</i>

Next lecture

- Light reflection on surfaces